

# Theory and Applications of Coupled Optical Waveguides Involving Anisotropic or Gyrotropic Materials

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**Abstract**—Coupled optical waveguides consisting of two isotropic dielectric slab waveguides coupled through anisotropic or gyrotropic materials inserted between them, are treated theoretically in detail. The properties of reciprocal and nonreciprocal TE-TM mode conversion and a nonreciprocal phase shift for TM modes are shown. As an example of application of this type of coupled waveguide, a nonreciprocal optical integrated circuit (IC) mode converter is proposed. It is shown that a circulator and an isolator which require neither mode separators nor mode filters can be constructed by utilizing the proposed nonreciprocal mode converter. The numerical design examples are also given.

## I. INTRODUCTION

OPTICAL integrated circuits (IC's) with anisotropic or gyrotropic materials are of great interest from both the theoretical and the practical points of view. So far, the electromagnetic wave modes propagating along an anisotropic or gyrotropic slab waveguide have been analyzed by several authors [1]–[3], and various optical IC devices such as mode converter, isolator, circulator, etc., using a waveguide involving an anisotropic or gyrotropic medium have been proposed [4]–[9].

In the present paper, the coupled optical waveguides, consisting of two parallel isotropic dielectric slab waveguides coupled through anisotropic or gyrotropic materials inserted between them, are considered. This is one of the simplest and most typical configurations of the optical IC involving an anisotropic or gyrotropic material. For example, an optical modulator using this type of coupled waveguide has been proposed previously [10]. To the authors' knowledge, however, the detailed analysis of the aforementioned coupled waveguides has not yet been reported so far.

In this paper, the general features of the wave propagation characteristics of this type of coupled waveguide are analyzed theoretically in detail, and the nonreciprocal optical IC mode converter is proposed as an example of application. It is shown that a circulator and an isolator requiring neither mode separators nor mode filters can be constructed utilizing the proposed nonreciprocal mode converter. The numerical design examples are also given.

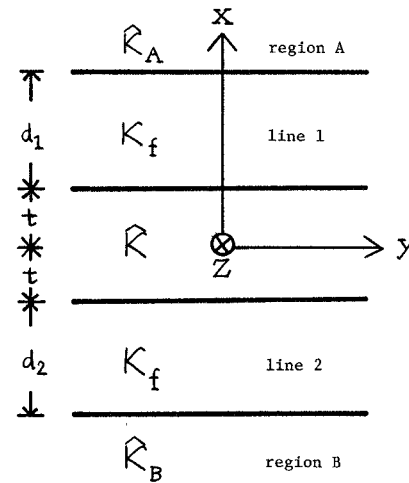


Fig. 1. Geometry of the coupled dielectric slab waveguides coupled through anisotropic or gyrotropic material.

## II. WAVEGUIDE CONFIGURATION AND THE BASIS OF ANALYSIS

### A. Waveguide Configuration

The coupled waveguide to be considered is a two-dimensional structure, as shown in Fig. 1, being uniform in the  $y$  direction. The direction of wave propagation is parallel to the  $z$  axis. Lines 1 and 2 are the isotropic dielectric slab waveguides whose permittivity is  $\epsilon_0 \kappa_f$  where  $\epsilon_0$  is the permittivity in vacuum. The medium between lines 1 and 2 is anisotropic or gyrotropic with tensor permittivity  $\epsilon_0 \hat{\kappa}$ . The permittivity tensors  $\epsilon_0 \hat{\kappa}_A$  and  $\epsilon_0 \hat{\kappa}_B$  in regions A and B, respectively, are assumed to be diagonal. It is assumed that the permeabilities in all regions are equal to the permeability in vacuum  $\mu_0$ . It is assumed further that the media involved are dissipation free.

The specific permittivity tensor  $\hat{\kappa}$  is expressed, in general, in the form

$$\hat{\kappa} = \begin{bmatrix} \kappa_{xx} & \kappa_{xy} & \kappa_{xz} \\ \kappa_{xy}^* & \kappa_{yy} & \kappa_{yz} \\ \kappa_{xz}^* & \kappa_{yz}^* & \kappa_{zz} \end{bmatrix} \quad (1)$$

where the asterisk denotes the complex conjugate. We shall distinguish the *anisotropic* material from the *gyrotropic* material in the following manner: The *anisotropic* material is a matter for which all the off-diagonal terms in  $\hat{\kappa}$  are real while the *gyrotropic* material is a matter for which all the off-diagonal terms in  $\hat{\kappa}$  are pure imaginary. The diagonal

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TABLE I  
CLASSIFICATION OF PERTURBED SYSTEMS

	Longitudinal(L)	Polar(P)	Equatorial(E)
$\hat{\kappa}$	$\begin{bmatrix} \kappa_{xx} & \kappa_{xy} & 0 \\ \kappa_{xy}^* & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{bmatrix}$	$\begin{bmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & \kappa_{yz} \\ 0 & \kappa_{yz}^* & \kappa_{zz} \end{bmatrix}$	$\begin{bmatrix} \kappa_{xx} & 0 & \kappa_{xz} \\ 0 & \kappa_{yy} & 0 \\ \kappa_{xz}^* & 0 & \kappa_{zz} \end{bmatrix}$
Anisotropic(A)	<b>A L</b> $\kappa_{xy}, \kappa_{xy}^*$ : real	<b>A P</b> $\kappa_{yz}, \kappa_{yz}^*$ : real	<b>A E</b> $\kappa_{xz}, \kappa_{xz}^*$ : real
Gyrotropic(G)	<b>G L</b> $\kappa_{xy}, \kappa_{xy}^*$ : pure imag. (z direction)*	<b>G P</b> $\kappa_{yz}, \kappa_{yz}^*$ : pure imag. (x direction)*	<b>G E</b> $\kappa_{xz}, \kappa_{xz}^*$ : pure imag. (y direction)*

(\* The direction of applied magnetic field ).

terms in  $\hat{\kappa}$  are real for both anisotropic and gyrotropic media. The anisotropic property can be realized by applying an electric field to the electrooptic materials, or rotating the crystal axis of the optical crystals. The gyrotropic property, on the other hand, can be obtained by applying a magnetic field to the magneto-optic materials.

### B. Basis of the Analysis

In most practical cases, the change of the material constants of electrooptic or magneto-optic materials caused by the applied electric or magnetic fields is very small. Therefore, the coupled waveguides system shown in Fig. 1 can be regarded as a slightly perturbed system, perturbed from an unperturbed system which possesses the diagonal permittivity tensor  $\epsilon_0 \hat{\kappa}'$  in place of  $\epsilon_0 \hat{\kappa}$ , where  $\hat{\kappa}'$  is given by

$$\hat{\kappa}' = \begin{bmatrix} \kappa_{xx} & 0 & 0 \\ 0 & \kappa_{yy} & 0 \\ 0 & 0 & \kappa_{zz} \end{bmatrix} \quad (\text{unperturbed}). \quad (2)$$

The nonzero off-diagonal terms in  $\hat{\kappa}$  are thought to appear as a result of slight perturbation caused by the applied electric or magnetic fields. It is assumed that the diagonal terms in  $\hat{\kappa}$  and the tensor permittivities  $\hat{\kappa}_A$  and  $\hat{\kappa}_B$  in regions A and B are not affected by the applied fields.

According to the positions of nonzero off-diagonal terms in  $\hat{\kappa}$ , the perturbed coupled waveguides system shown in Fig. 1 can be classified in three categories [7]. These are longitudinal (L), polar (P), and equatorial (E) perturbations as shown in Table I. Further, each perturbation is divided into anisotropic (A) and gyrotropic (G) types. Consequently, we have to treat the wave propagation characteristics about the aforementioned six perturbed systems.

Let us consider now two individual basic waveguide systems as shown in Fig. 2(a) and (b). This figure shows the basic configurations of the individual lines 1 and 2. The medium of permittivity tensor  $\epsilon_0 \hat{\kappa}'$  extends in the regions of  $x \leq -t$  and  $x \geq t$  for the basic configurations of the individual lines 1 and 2, respectively. Hereafter, we shall refer to the guided modes propagating in these two individual basic waveguide systems as elementary modes.

Fig. 3 illustrates numerically the propagation constants of the lowest two elementary modes of lines 1 and 2.

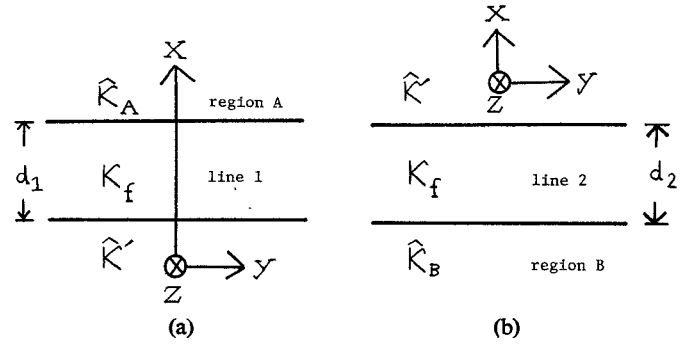


Fig. 2. Geometry of two individual basic lines. (a) The basic configuration of line 1. (b) The basic configuration of line 2.

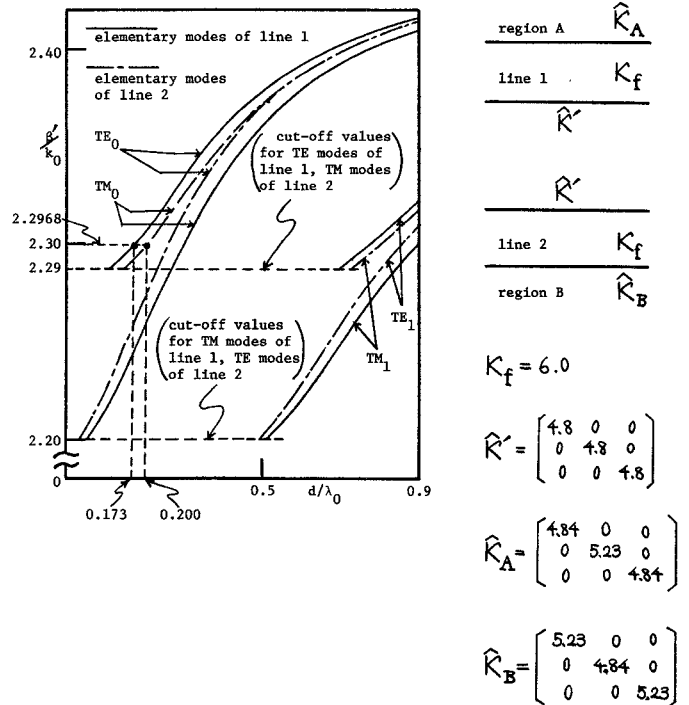


Fig. 3. Dispersion curves for the lowest two elementary modes propagating along each individual basic line.

The values of the permittivities in the regions used in the numerical calculations are shown in the same figure. The values used as a numerical example correspond to the following materials: The slab waveguide consists of  $\text{As}_2\text{S}_3$  [11], the media in regions A and B are  $\text{TeO}_2$  [12], and the medium between two lines is assumed to be  $\text{K}_3\text{Li}_2\text{Nb}_5\text{O}_{15}$  [13] as an anisotropic material and YIG [6] as a gyrotropic material. Operation wavelength is assumed to be  $\lambda_0 \approx 1.152 \mu\text{m}$ . The ordinate is a propagation constant  $\beta'$  normalized by the propagation constant in free space  $k_0 (= \omega \sqrt{\epsilon_0 \mu_0})$ , and the abscissa is a film thickness  $d_1$ , or  $d_2$ , of the line 1, or line 2, normalized by a free-space wavelength  $\lambda_0 (= 2\pi/k_0)$ .

Since the permittivity tensor  $\hat{\kappa}'$  is diagonal, TE and TM modes of propagation can be supported in the basic systems. The normalized propagation constant  $\beta'/k_0$  of the guided modes in a dielectric slab waveguide possesses two limiting

values. The upper limit is determined by the refractive index of the film, whereas the lower limit is determined by either one of the refractive indices of the surrounding media whose refractive index is larger than the other. In the case of our numerical example, the lower limits of the normalized propagation constant of the elementary modes in two basic systems are different for the TE modes and the TM modes as we can see from Fig. 3. The TE modes of line 1 are cut off at  $\beta'/k_0 = \sqrt{5.23} = 2.29$  while the TM modes at  $\beta'/k_0 = \sqrt{4.84} = 2.20$ . Similarly, the TE modes of line 2 are cut off at 2.20 and the TM modes at 2.29. By choosing the film thicknesses  $d_1, d_2$  of lines 1 and 2 appropriately, the elementary modes of lines 1 and 2 can be degenerated. The points in Fig. 3 (i.e.,  $d_1/\lambda_0 = 0.173$  and  $d_2/\lambda_0 = 0.200$ ) show the example of the film thicknesses for which the dominant TE mode (TE<sub>0</sub>) of line 1 and the dominant TM mode (TM<sub>0</sub>) of line 2 are degenerated.

### III. ANALYSIS BASED ON THE VARIATIONAL METHOD

#### A. Normal Mode Analysis of the Perturbed Systems

Since the off-diagonal terms in  $\hat{K}$  which appeared as a result of the perturbation are actually very small in most practical cases, the electromagnetic fields of the normal modes propagating along the perturbed systems can be expressed approximately in terms of the linear combinations of the elementary modes of lines 1 and 2.

Let us suppose now that, by choosing the film thicknesses  $d_1$  and  $d_2$  appropriately, the TE<sub>0</sub> mode of line 1 and the TM<sub>0</sub> mode of line 2 are degenerated or nearly degenerated, and the coupling between these two degenerated modes and other modes is small enough to be negligible. Then the field distributions of the normal mode in the perturbed system can be expressed by the linear combinations of the electric and magnetic fields of the elementary modes TE<sub>0</sub> and TM<sub>0</sub>. Hence, applying the variational technique given in [14], the sufficiently precise value of the propagation constant and the field distributions of the normal mode in the perturbed systems can be obtained. If the spacing between lines 1 and 2 is very small, or the difference between the refractive indices of the film and the intermedium between the two lines is very small, the accuracy of the aforementioned method of analysis becomes poor. This will be discussed in Section III-B with numerical examples.

Let the normalized complex amplitudes of the TE<sub>0</sub> mode of line 1 and the TM<sub>0</sub> mode of line 2 be  $a_1(z)$  and  $a_2(z)$ , respectively. Then, the complex amplitudes  $a_1(z)$  and  $a_2(z)$  can be expressed in terms of the complex amplitudes  $a_1(0)$  and  $a_2(0)$  at the beginning of the coupling portion  $z = 0$  as follows:

$$\begin{bmatrix} a_1(z) \\ a_2(z) \end{bmatrix} = [S]^F \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix} [\exp(-j\beta_0 z)] \quad (3)$$

where

$$[S]^F = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \quad (4)$$

and

$$\beta_0 = (\beta_1 + \beta_2)/2$$

where  $\beta_1$  and  $\beta_2$  are the propagation constants of the elementary modes TE<sub>0</sub> and TM<sub>0</sub> of lines 1 and 2, respectively. The matrix  $[S]^F$  given by (4) can be regarded as a scattering matrix with respect to the waves propagating in the positive  $z$  direction, i.e., the forward waves. We will derive in Section IV-A the scattering matrix which describes the whole system including both forward and backward waves.

*L and P Perturbations:* The matrix elements in  $[S]^F$  are given by

$$\begin{aligned} S_{11} &= \cos cz - j \Delta \sin cz \\ S_{12} &= -j(1/\alpha)\sqrt{1 - \Delta^2} \sin cz \\ S_{21} &= -j\alpha\sqrt{1 - \Delta^2} \sin cz \\ S_{22} &= \cos cz + j \Delta \sin cz \end{aligned} \quad (5)$$

where

$$C = \beta_0 F \sqrt{1 + (\delta/F)^2} \quad \delta = (\beta_1 - \beta_2)/(\beta_1 + \beta_2)$$

$$\Delta = (\delta/F)/\sqrt{1 + (\delta/F)^2} \quad F = |N_{12}'|/4\beta_0$$

$$N_{ij}' = \omega \epsilon_0 \int_{-\infty}^{\infty} \mathbf{e}_i \cdot (\hat{\mathbf{K}} - \hat{\mathbf{K}}') \mathbf{e}_j dX \quad (i, j = 1, 2). \quad (6)$$

$N_{ij}'$  is regarded as a perturbation term in which  $\mathbf{e}_1$  and  $\mathbf{e}_2$  denote the electric field vectors of the TE<sub>0</sub> mode of line 1 and the TM<sub>0</sub> mode of the line 2, respectively. For each perturbation, the coefficient  $\alpha$  in the equations given by (5) becomes as follows:

$$\alpha = \begin{cases} +1, & (N_{12}' > 0) \\ -1, & (N_{12}' < 0) \end{cases} \quad (7)$$

for *AL* and *GP* perturbations, and

$$\alpha = \begin{cases} +j, & (\tilde{N}_{12} > 0) \\ -j, & (\tilde{N}_{12} < 0) \end{cases} \quad (8)$$

for *GL* and *AP* perturbations, where  $\tilde{N}_{12}'$  in (8) is a real quantity defined as such  $N_{12}' = j\tilde{N}_{12}'$  and  $j = \sqrt{-1}$ .

*GE Perturbation:* The scattering matrix defined by (3) becomes diagonal in the form

$$[S]^F = \begin{bmatrix} \exp(-j\delta\beta_0 Z) & 0 \\ 0 & \exp[j(\delta\beta_0 - \Delta\beta_2)Z] \end{bmatrix} \quad (9)$$

where

$$\Delta\beta_2 = N_{22}'/D_{22}. \quad (10)$$

In the case of *L*- and *P*-perturbation systems, the coupling may occur between TE<sub>0</sub> and TM<sub>0</sub> modes since  $S_{12} \neq 0$  and  $S_{21} \neq 0$  as shown in (5). Further, by comparing *AL* perturbation with *GL* perturbation, it can be seen that the TM<sub>0</sub> mode output is 90° out of phase with respect to the TE<sub>0</sub> mode. Similarly, by comparing *GP* perturbation with *AP* perturbation, we found that the phase of the TE<sub>0</sub> mode output is different in 90° from that of the TM<sub>0</sub> mode.

In the case of *GE* perturbation, no mode conversion between TE<sub>0</sub> and TM<sub>0</sub> modes can be expected, and the TM<sub>0</sub> mode suffers additional phase shift due to the perturbation in the amount of  $\Delta\beta_2 z$ . *AE* perturbation, on the other hand, causes no influence on both TE<sub>0</sub> and TM<sub>0</sub>

modes. We shall omit, therefore, the case of the  $AE$  perturbation in the following considerations.

In order to discuss the wave propagation characteristics such as, for instance, the nonreciprocal properties of the perturbed systems, the backward waves must be taken into consideration as well as the forward waves.

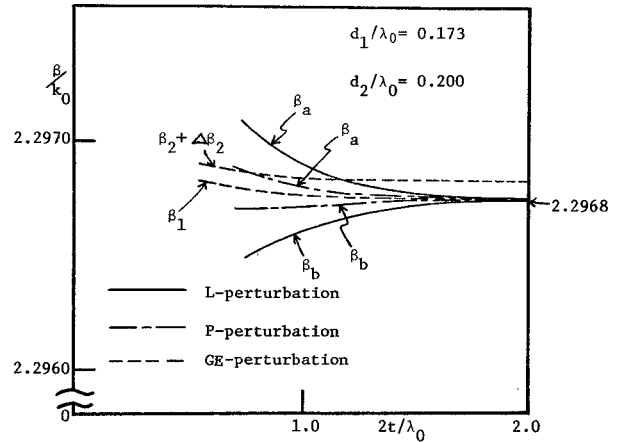
The scattering matrices  $[S]^B$  for the backward waves can be derived by a similar manner as that described in [7]. In the result, for both  $AL$ - and  $GL$ -perturbation systems, the scattering matrices  $[S]^B$  for the backward waves become entirely the same as  $[S]^F$  for the forward waves, while in  $P$ -perturbation systems, the signs of off-diagonal terms  $S_{12}$  and  $S_{21}$  are reversed, and in  $GE$ -perturbation systems, the sign of  $\Delta\beta_2$  is reversed.

By comparing the matrices  $[S]^B$  with that for the forward waves given by (4) and (9), we find the following features of the wave propagation characteristics of the perturbed systems: The  $L$ -perturbation systems are bilateral, since the scattering matrices are invariant regardless of the direction of the wave propagation. On the contrary, in the  $P$ -perturbation systems, the phase shift for the  $TM_0(TE_0)$  mode output relative to the  $TE_0(TM_0)$  mode input shows  $180^\circ$  difference between forward and backward directions of propagation. In the  $EG$ -perturbation system, on the other hand, the phase of the  $TM_0$  mode lags by  $\Delta\beta_2 z$  in the forward direction, while it leads by  $\Delta\beta_2 z$  in the backward direction, both comparing with that in the unperturbed systems.

### B. Numerical Example

Fig. 4 illustrates numerically the normalized propagation constants  $\beta/k_0$  of the normal modes propagating in the forward direction in  $L$ -,  $P$ -, and  $GE$ -perturbation systems. The film thicknesses  $d_1$  and  $d_2$  of line 1 and line 2 were chosen as  $d_1/\lambda_0 = 0.173$  and  $d_2/\lambda_0 = 0.200$  so that the  $TE_0$  mode of line 1 and  $TM_0$  mode of line 2 are degenerated as shown in Fig. 3. In our example, the  $TM_0$  mode of line 1 and  $TE_0$  mode of line 2 can also be propagated. However, the coupling between these two modes and the aforementioned degenerated  $TE_0$  mode of line 1 and  $TM_0$  mode of line 2 is weak enough to be negligible because the differences of propagation constants between these two mode groups are sufficiently large (about  $10^2$  times) compared with the perturbation terms as we can see from Fig. 3. It can be shown further that, in our numerical example, the proposed approximate method of analysis using only two elementary modes is certified with satisfactory accuracy as long as the spacing  $2t$  between two lines is greater than  $\lambda_0/2$  (i.e.,  $2t/\lambda_0 > 0.5$ ). However, in order to obtain the sufficiently strong coupling, it is desirable to choose the spacing  $2t$  smaller than about  $1.5\lambda_0$  (i.e.,  $2t/\lambda_0 < 1.5$ ).

We can see from Fig. 4 that the normalized propagation constants of the normal modes in the  $L$ - and  $P$ -perturbation systems approach that of the degenerated elementary modes,  $\beta_1/k_0 = \beta_2/k_0 = 2.2968$ , as increasing the spacing between two lines. In the  $GE$ -perturbation system, on the other hand, either one of the propagation constants varies in the amount of  $\Delta\beta_2$  due to the perturbation.



$$\begin{aligned}
 \text{L-perturbation: } & \begin{cases} \hat{K} = \begin{bmatrix} 4.8 & K_{xy} & 0 \\ K_{xy}^* & 4.8 & 0 \\ 0 & 0 & 4.8 \end{bmatrix} \\ |K_{xy}| = 0.05 \end{cases} \\
 \text{P-perturbation: } & \begin{cases} \hat{K} = \begin{bmatrix} 4.8 & 0 & 0 \\ 0 & 4.8 & K_{yz} \\ 0 & K_{yz}^* & 4.8 \end{bmatrix} \\ |K_{yz}| = 0.05 \end{cases} \\
 \text{EG-perturbation: } & \hat{K} = \begin{bmatrix} 4.8 & 0 & j0.005 \\ 0 & 4.8 & 0 \\ -j0.005 & 0 & 4.8 \end{bmatrix}
 \end{aligned}$$

Fig. 4. Dispersion curve for forward normal modes of  $L$ -,  $P$ -, and  $GE$ -perturbation systems.

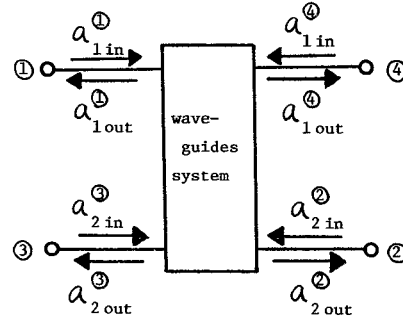


Fig. 5. Circuit-theoretic model for the coupled waveguide system.

## IV. TREATMENT OF THE PERTURBED WAVEGUIDE SYSTEMS USING A SCATTERING MATRIX

### A. Scattering Matrix for the Whole System

Let us take only two degenerated elementary modes into consideration as mentioned in the preceding example. If we assign one port to each input end of particular forward and backward elementary modes, the perturbed waveguide system can be expressed as shown in Fig. 5. The ports ① and ④ denote the input ends of the forward and backward  $TE_0$  mode of line 1, respectively, while the ports ③ and ② represent the input ends of the forward and backward  $TM_0$

mode of line 2, respectively.  $a_1(z)$  and  $a_2(z)$  are the normalized complex amplitudes of the TE<sub>0</sub> mode of line 1 and TM<sub>0</sub> mode of line 2, respectively. A superscript  $(i)$  ( $i = 1, 2, 3, 4$ ) denotes the port number, and the subscripts "in" and "out" mean input and output, respectively. The scattering matrix describing the whole system can then be expressed as

$$\begin{aligned} a_{\text{out}} &= \exp(-j\beta_0 l) [S] \cdot a_{\text{in}} \\ [S] &= \begin{bmatrix} 0 & [S]^B \\ [S]^F & 0 \end{bmatrix} \end{aligned} \quad (11)$$

where  $[S]^F$  and  $[S]^B$  are the scattering matrices for the forward and backward waves, respectively, obtained in the preceding section,  $l$  denotes the length of the system, and

$$\begin{aligned} a_{\text{out}} &= T(a_{1\text{out}}^{(1)}, a_{2\text{out}}^{(3)}, a_{1\text{out}}^{(2)}, a_{2\text{out}}^{(4)}) \\ a_{\text{in}} &= T(a_{1\text{in}}^{(1)}, a_{2\text{in}}^{(3)}, a_{1\text{in}}^{(2)}, a_{2\text{in}}^{(4)}) \\ 0 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}. \end{aligned} \quad (12)$$

In (12),  $T$  denotes the transpose. Since the system is assumed to be dissipation free, the scattering matrix  $[S]$  is unitary.

The system is reciprocal provided that the scattering matrix is symmetrical. On the contrary, if

$$[S]^F \neq T[S]^B \quad (13)$$

the system becomes nonreciprocal. The reciprocal and non-reciprocal properties of the perturbed waveguide systems can then be summarized as follows.

**L perturbations:** The AL-perturbation system is reciprocal, but the GL-perturbation system is nonreciprocal.

**P perturbations:** The AP-perturbation system is reciprocal, whereas the GP-perturbation system is nonreciprocal.

**GE perturbation:** The GE-perturbation system is non-reciprocal.

### B. Application to a Nonreciprocal Mode Converter

As stated in the preceding section, the AL- and AP-perturbation systems show the reciprocal mode conversion while the GL- and GP-perturbation systems possess the property of nonreciprocal mode conversion, and the GE-perturbation system presents the nonreciprocal phase shift.

As an example of application of these perturbed systems, let us consider the nonreciprocal optical IC mode converter consisting of the GL-AP combination system as shown in Fig. 6. Again we shall assume that the TE<sub>0</sub> mode of line 1 and the TM<sub>0</sub> mode of line 2 are degenerated. In this case,  $\Delta$  given by (6) vanishes. Let us designate  $c$  in (5) as  $c_{GL}$  and  $c_{AP}$  for the GL and AP perturbations, respectively. Let us assume also that the lengths of the GL- and AP-perturbation systems are so chosen that

$$c_{GL}Z = \pi/4 \quad c_{AP}Z = \pi/4. \quad (14)$$

Then, the scattering matrix for the forward wave is yielded by multiplying the matrices for the GL- and AP-perturba-

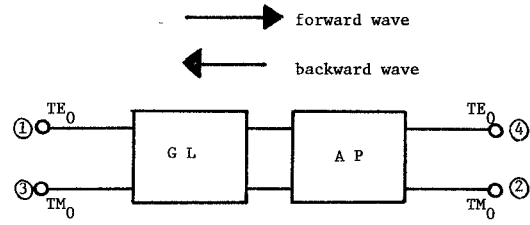
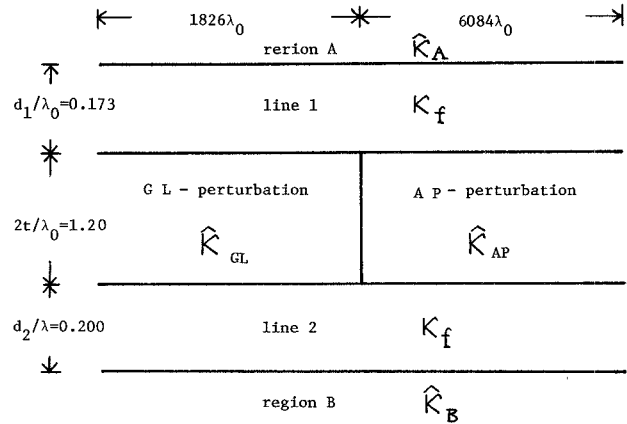


Fig. 6. Nonreciprocal mode converter consisting of GL-AP combination system.



$$K_f = 6.0, \quad \hat{K}_{GL} = \begin{bmatrix} 4.8 & j0.05 & 0 \\ -j0.05 & 4.8 & 0 \\ 0 & 0 & 4.8 \end{bmatrix}$$

$$\hat{K}_{AP} = \begin{bmatrix} 4.8 & 0 & 0 \\ 0 & 4.8 & 0.05 \\ 0 & 0.05 & 4.8 \end{bmatrix}, \quad \hat{K}_A = \begin{bmatrix} 4.84 & 0 & 0 \\ 0 & 5.23 & 0 \\ 0 & 0 & 4.84 \end{bmatrix}$$

$$\hat{K}_B = \begin{bmatrix} 5.23 & 0 & 0 \\ 0 & 4.84 & 0 \\ 0 & 0 & 5.23 \end{bmatrix}$$

Fig. 7. Numerical design example of circulator using GL-AP combination system.

tion systems given by (4) as follows:

$$[S]^F = [S]_{AP}^F \cdot [S]_{GL}^F = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}. \quad (15)$$

Similarly, the scattering matrix for the backward wave is given by

$$[S]^B = [S]_{GL}^B \cdot [S]_{AP}^B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (16)$$

It can be seen from (15) and (16) that this system acts as a circulator rotating the port  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4$ . Fig. 7 shows the numerical design example of this circulator.

Unlike conventional optical circulators proposed previously, this circulator requires no mode separators at both input and output ports. Furthermore, the line 1 or line 2 of this system can also be utilized as an isolator for the TE<sub>0</sub> mode or the TM<sub>0</sub> mode, respectively, without using the mode filters at both input and output ports.

Besides this example, a wide variety of reciprocal and

nonreciprocal optical IC devices possessing various functions would be constructed by using the perturbed systems treated in the present paper.

### V. CONCLUSION

The coupled optical waveguides, consisting of two isotropic dielectric slab waveguides coupled through anisotropic or gyrotropic materials inserted between them, have been treated theoretically in detail. It has been found that the *AL*- and *AP*-perturbation systems show the reciprocal mode conversion, while the *GL*- and *GP*-perturbation systems possess the property of nonreciprocal mode conversion, and the *GE*-perturbation system causes nonreciprocal phase shift for the  $TM_0$  mode, whereas the *AE*-perturbation system shows no influence upon both  $TE_0$  and  $TM_0$  modes. As an example of application of these perturbed systems, the nonreciprocal optical IC mode converter has been proposed, and the numerical design example of the optical IC circulator has been given. This circulator requires no mode separators at both input and output ports. This circulator can also be utilized as an isolator without using mode filters at both input and output ports. In order to realize these devices, the progress of the fabrication techniques, together with the development of magneto-optic materials which possess a large Faraday effect, is required.

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# Linear Power Responses of an Optical Fiber

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**Abstract**—It is known that an optical fiber behaves linearly in terms of power when the modulation frequency is smaller than the spectrum width of the light source. In order to calculate the impulse or frequency power responses with a modal calculation, it is shown that the powers carried by the different modes are independent in usual cases. Different formulas are proposed for the linear responses when there is no mode coupling, and the corresponding validity conditions are given.

### I. INTRODUCTION

A DESIRABLE characteristic of any transmission system is the linear relation between the output and input variables. In the case of transmission through optical fibers, the output variable is the current generated by the

photodetector, and it is proportional to the optical power. Then the fiber must be linear in terms of power. Some aspects of this linearity have already been studied [1], [2]. It may be obtained by using an incoherent source of spectral width  $\Delta\nu$  when the modulation frequencies are quite lower than  $\Delta\nu$  [1].

A modal calculation of the impulse and frequency power responses, when there is no mode coupling, is proposed (Section III). But before exposing our results, we must justify the validity of such a method (Section II).

### II. DO DIFFERENT MODES CARRY INDEPENDENT CONTRIBUTIONS TO THE GUIDED POWER?

It is commonly assumed that the answer is positive.

Since powers of unmodulated modes are independent in case of lossless guides only, we shall consider our fiber as a